GSE ADVANCED ALGEBRA/ALGEBRA II

11

SCOPE AND SEQUENCE CHART

Unit Name	Unit Description	Georgia Standards of Excellence	Unit Duration
1 Quadratics Revisited	**Unit1**: Students will revisit solving quadratic equations in this unit. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. Students will perform operations with complex numbers and solve quadratic equations with complex solutions. Students will also extend the laws of exponents to rational exponents and use those properties to evaluate and simplify expressions containing rational exponents.	**Perform arithmetic operations with complex numbers.** **MGSE9-12.N.CN.1** Understand there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi where a and b are real numbers. **MGSE9-12.N.CN.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **MGSE9-12.N.CN.3** Find the conjugate of a complex number; use the conjugate to find the ~~absolute value (modulus) and~~ quotient of complex numbers. **Use complex numbers in polynomial identities and equations.** **MGSE9-12.N.CN.7** Solve quadratic equations with real coefficients that have complex solutions by (but not limited to) square roots, completing the square, and the quadratic formula. **MGSE9-12.N.CN.8** Extend polynomial identities to include factoring with complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$. **Solve equations and inequalities in one variable** **MGSE9-12.A.REI.4** Solve quadratic equations in one variable. **MGSE9-12.A.REI.4b** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square, and the quadratic formula, as appropriate to the initial form of the equation ~~(limit to real number solutions)~~.	4 – 5 Weeks

		Extend the properties of exponents to <u>rational exponents.</u> **MGSE9-12.N.RN.1** Explain how the meaning of rational exponents follows from extending the properties of integer exponents to rational numbers, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{(1/3)}$ *to be the cube root of 5 because we want* $[5^{(1/3)}]^3 = 5^{[(1/3)\,x\,3]}$ *to hold, so* $[5^{(1/3)}]^3$ *must equal 5.* **MGSE9-12.N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.	
2 **Operations with Polynomials**	**<u>Unit 2</u>**: This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will find inverse functions and verify by composition that one function is the inverse of another function.	**<u>Perform arithmetic operations on polynomials</u>** **MGSE9-12.A.APR.1** Add, subtract, and multiply polynomials; understand that polynomials form a system analogous to the integers in that they are closed under these operations. **MGSE9-12.A.APR.5** Know and apply that the Binomial Theorem gives the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. **<u>Rewrite rational expressions</u>** **MGSE9-12.A.APR.**6 Rewrite simple rational expressions in different forms using inspection, long division, or a computer algebra system; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x). **<u>Build a function that models a relationship between two quantities</u>** **MGSE9-12.F.BF.1** Write a function that describes a relationship between two quantities. **MGSE9-12.F.BF.1b** Combine standard function types using arithmetic operations in contextual situations (Adding, subtracting, and multiplying functions of different types). **MGSE9-12.F.BF.1c** Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.* **<u>Build new functions from existing functions</u>**	4 – 5 Weeks

		MGSE9-12.F.BF.4 Find inverse functions. **MGSE9-12.F.BF.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2(x^3)$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. **MGSE9-12.F.BF.4b** Verify by composition that one function is the inverse of another. **MGSE9-12.F.BF.4c** Read values of an inverse function from a graph or a table, given that the function has an inverse.	
3 Polynomial Functions	**Unit 3**: In this unit, students continue their study of polynomials by identifying zeros and making connections between zeros of a polynomial and solutions of a polynomial equation. Students will see how the Fundamental Theorem of Algebra can be used to determine the number of solutions of a polynomial equation and will find all the roots of those equations. Students will graph polynomial functions and interpret the key characteristics of the function.	**MGSE9-12.N.CN.9** Use the Fundamental Theorem of Algebra to find all roots of a polynomial equation **Interpret the structure of expressions MGSE9-12.A.SSE.1** Interpret expressions that represent a quantity in terms of its context. **MGSE9-12.A.SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients, in context. **MGSE9-12.A.SSE.1b** Given situations which utilize formulas or expressions with multiple terms and/or factors, interpret the meaning (in context) of individual terms or factors. **MGSE9-12.A.SSE.2** Use the structure of an expression to rewrite it in different equivalent forms. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. **Understand the relationship between zeros and factors of polynomials** **MGSE9-12.A.APR.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **MGSE9-12.A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Use polynomial identities to solve problems** **MGSE9-12.A.APR.4** Prove polynomial identities and use them to	4 – 5 Weeks

		describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.* **Interpret functions that arise in applications in terms of the context** **MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. **Analyze functions using different representations** **MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. **MGSE9-12.F.IF.7c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	
4 **Rational & Radical Relationships**	**Unit 4**: Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this unit is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. Similarly, radical expressions follow the rules governed by irrational numbers.	**Rewrite rational expressions** **MGSE9-12.A.APR.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **Create equations that describe numbers or relationships** **MGSE9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from ~~linear, quadratic~~, simple rational, ~~and exponential~~ functions (~~integer inputs only~~). **MGSE9-12.A.CED.2** Create ~~linear, quadratic, and exponential~~ equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. *(Limit to rational and radical functions. The phrase "in two or more variables" refers to formulas like the compound interest formula, in which* $A = P(1 + r/n)^{nt}$ *has multiple variables.)* **Understand solving equations as a process of reasoning and explain the reasoning**	5 – 6 Weeks

		MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **MGSE9-12.F.IF.4** Using tables, graphs, and verbal descriptions, interpret the key characteristics of a function which models the relationship between two quantities. Sketch a graph showing key features including: intercepts; interval where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. **Interpret functions that arise in applications in terms of the context** **MGSE9-12.F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.* **Analyze functions using different representations** **MGSE9-12.F.IF.7** Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. **MGSE9-12.F.IF.7b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. **MGSE9-12.F.IF.7d** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	
5 Exponential & Logarithms	**Unit 5**: Students extend their work with exponential functions to include solving exponential equations with logarithms. They analyze the relationship between these two functions.	**Write expressions in equivalent forms to solve problems** **MGSE9-12.A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. **MGSE9-12.A.SSE.3c** Use the properties of exponents to transform expressions for exponential functions. *For example, the expression* 1.15^t *, where t is in years, can be rewritten as* $[1.15^{(1/12)}]^{(12t)} \approx 1.012^{(12t)}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.* **Analyze functions using different representations**	4 – 5 Weeks

		MGSE9-12.F.IF.7 Graph functions expressed algebraically and show key features of the graph both by hand and by using technology. **MGSE9-12.F.IF.7e** Graph exponential and logarithmic functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude.~~ **MGSE9-12.F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. **MGSE9-12.F.IF.8b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{(12t)}$, $y = (1.2)^{(t/10)}$, *and classify them as representing exponential growth and decay.* **Build new functions from existing functions** **MGSE9-12.F.BF.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **Construct and compare linear, quadratic, and exponential models and solve problems** **MGSE9-12.F.LE.4** For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	
6 Mathematical Modeling	**Unit 6**: In this unit students synthesize and generalize what they have learned about a variety of function families. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. They determine whether it is best to model with multiple functions creating a piecewise function. Students will also explore	**Write expressions in equivalent forms to solve problems** **MGSE9-12.A.SSE.4** Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.* **MGSE9-12.A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, simple rational, and exponential functions ~~(integer inputs only)~~. **MGSE9-12.A.CED.2** Create linear, quadratic, and exponential equations in two or more variables to represent relationships between	5 – 6 Weeks

	the sum of finite geometric series.	quantities; graph equations on coordinate axes with labels and scales. *(The phrase "in two or more variables" refers to formulas like the compound interest formula, in which $A = P(1 + r/n)^{nt}$ has multiple variables.)* **MGSE9-12.A.CED.3** Represent constraints by equations or inequalities, and by systems of equation and/or inequalities, and interpret data points as possible (i.e. a solution) or not possible (i.e. a non-solution) under the established constraints. **MGSE9-12.A.CED.4** Rearrange formulas to highlight a quantity of interest using the same reasoning as in solving equations. *Examples: Rearrange Ohm's law V = IR to highlight resistance R; Rearrange area of a circle formula $A = \pi r^2$ to highlight the radius r.* **Represent and solve equations and inequalities graphically** **MGSE9-12.A.REI.11** Using graphs, tables, or successive approximations, show that the solution to the equation f(x) = g(x) is the x-value where the y-values of f(x) and g(x) are the same. **Interpret functions that arise in applications in terms of the context** **MGSE9-12.F.IF.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **MGSE9-12.F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one function and an algebraic expression for another, say which has the larger maximum.* **Build new functions from existing functions** **MGSE9-12.F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	
7 Inferences and Conclusions from Data	**Unit 7**: In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that	**Summarize, represent, and interpret data on a single count or measurement variable** **MGSE9-12.S.ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, ~~mean absolute deviation~~, standard deviation) of two or more	4 – 5 Weeks

	randomness and careful design play in the conclusions that can be drawn.	different data sets. **MGSE9-12.S.ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. **Understand and evaluate random processes underlying statistical experiments** **MGSE9-12.S.IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. **MGSE9-12.S.IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0. 5. Would a result of 5 tails in a row cause you to question the model?* **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** **MGSE9-12.S.IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **MGSE9-12.S.IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **MGSE9-12.S.IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **MGSE9-12.S.IC.6** Evaluate reports based on data. *For example, determining quantitative or categorical data; collection methods; biases or flaws in data.*	